## STEPHEN F. AUSTIN STATE UNIVERSITY

### **Department of Mathematics and Statistics**

# Math 539 - Real Variables I Course Syllabus

<u>Course description:</u> A brief review of set theory is followed by a detailed study of metric spaces, normed linear spaces, and inner product spaces. Topics such as open and closed sets along with compactness and completeness are studied within the context of these spaces.

### **Credit hours:** 3

Course Prerequisites and Corequisites: MTH 440

**Course outline:** This is the first part of the two part introductory course in Real Analysis. This course introduces the fundamental concepts and topics in Real Analysis. In this course we cover the topics listed below:

Approximate time spent

| low: |  | Approximate time spen |
|------|--|-----------------------|
| •    | Elementary Set Theory  | 10%                   |
|      | <ul> <li>Basic definitions and operations on sets</li> </ul>                   |                       |
|      | <ul> <li>Function image sets and preimages</li> </ul>                          |                       |
|      | <ul> <li>Equivalence relations</li> </ul>                                      |                       |
| •    | Countable and Uncountable Sets   | 10%                   |
|      | <ul> <li>Definitions and theorems of countable and uncountable sets</li> </ul> |                       |
|      | o Powers sets  |                       |
|      | <ul> <li>Uncountability of the real numbers</li> </ul>                         |                       |
| •    | Metric Spaces  | 15%                   |
|      | <ul> <li>Definitions, theorems, and examples of metric spaces</li> </ul>       |                       |
|      | <ul> <li>Convergence and limits in metric spaces</li> </ul>                    |                       |
|      | <ul> <li>Open, closed, and dense sets</li> </ul>                               |                       |
| •    | Complete Metric Spaces   | 10%                   |
|      | <ul> <li>Definitions and examples</li> </ul>                                   |                       |
|      | <ul> <li>Completion of a metric space.</li> </ul>                              |                       |
| •    | Compactness in Metric Spaces   | 15%                   |
|      | <ul> <li>Definitions and examples</li> </ul>                                   |                       |
|      | <ul> <li>Totally bounded sets, Heine Borel</li> </ul>                          |                       |
| •    | Vector Spaces  | 10%                   |
|      | <ul> <li>Definitions and examples</li> </ul>                                   |                       |
|      | <ul> <li>Subspaces</li> </ul>  |                       |
|      | <ul> <li>Functionals</li> </ul>  |                       |
| •    | Normed Linear Spaces   | 10%                   |
|      | <ul> <li>Definitions and examples</li> </ul>                                   |                       |
|      | <ul> <li>Banach space</li> </ul>   |                       |
|      | <ul> <li>Metric induced by a norm</li> </ul>                                   |                       |
|      | <ul> <li>Equivalence of norms</li> </ul>                                       |                       |
| •    | Inner Product Spaces   | 20%                   |
|      | <ul> <li>Definition and examples</li> </ul>                                    |                       |
|      | <ul> <li>Hilbert space</li> </ul>  |                       |
|      | <ul> <li>Orthogonality</li> </ul>  |                       |
|      | Separable Hilbert space  |                       |
|      | Riesz-Fischer Theorem  |                       |
|      | Orthogonal complement  |                       |
|      | Parallelogram law  |                       |
|      | <ul> <li>Characterization of inner product spaces</li> </ul>                   |                       |

Student Learning Outcomes (SLO): At the end of MTH 539, a student who has studied and learned the material should be able to:

- 1. Define and apply basic set theoretic notions, including function image sets and preimage sets, cardinality, and countability. [MTH-PLO: 1,2,3,5], [STA-PLO: n.a.]
- 2. Understand and apply the basic properties and concepts of abstract metric spaces, especially complete metric spaces and convergence in metric spaces. [MTH-PLO: 1,2,3,5], [STA-PLO: n.a.]
- 3. Prove and apply the standard results for Banach and Hilbert spaces. [MTH-PLO: 1,2,3,5], [STA-PLO: n.a.]

### **Program Learning Outcomes (MTH - PLO):**

Students graduating from SFASU with a M.S. degree and a major in mathematics will:

- 1. **[Critical Reasoning]** Independently apply the principles of logic in mathematics to develop and analyze conjectures and proofs. (understanding of abstract structures, development of definitions, development and proof of conjectures)
- 2. **[Skills]** Execute advanced mathematical procedures and build upon these standard procedures. (learning of new skills, applying or extending skills in new situations)
- 3. **[Concepts]** Demonstrate knowledge of core mathematical concepts. (definitions and theorems in analysis, definitions and theorems in linear or abstract algebra, definitions and theorems in theoretical statistics)
- 5. **[Communication]** Demonstrate proficiency in communicating mathematics in a format appropriate to expected audiences. (written, visual, oral)

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