Stephen F. Austin State University

Department of Mathematics and Statistics

Math 512 – Abstract Algebra II
Course Syllabus

Course description: Rings and ideals, vector spaces, fields, integral domains, introduction to Galois Theory.

Credit hours: 3

Course Prerequisites and Corequisites: MTH 511

Course outline:

Topic	Approximate time spent
Introduction to Ring Theory	15%
Integral Domains	15%
Polynomial Rings and Factorization	10%
Ring Homomorphisms	15%
Vector Spaces	15%
Field extensions	15%
Finite Fields	5%
Galois Theory	10%

- **Introduction to Ring Theory** — 15%
    - Definition and examples of ring
    - Definition and examples of integral domain
    - Definition and examples of field
    - Ideals
        - Primes ideals
        - Maximal ideals
        - Principal ideals
    - Factor rings
    - Field of quotients
- **Integral Domains** — 15%
    - Cancellation
    - Unique Factorization Domains
    - Principal Ideal Domains
    - Euclidean Domains
    - Theorems relating different types of domains
- **Polynomial Rings and Factorization** — 10%
    - Irreducibility and Complete reducibility
    - Characteristic of a polynomial
    - Induced properties of polynomial rings over Field, Domains, etc.
- **Ring Homomorphisms** — 15%
    - Definition and intuition of ring homomorphism
    - Basic properties
    - Tests for proving homomorphism and isomorphism
    - First Isomorphism Theorem for Rings
    - [Commutative Diagrams]
- **Vector Spaces** — 15%
    - Linear algebra definitions in an algebraic context
    - Linear independence
- **Field extensions** — 15%
    - The Fundamental Theorem of Field Theory
    - Splitting Fields
    - Algebraic Extensions
    - Finite Extensions
- **Finite Fields** — 5%
    - Classification of Finite Fields
    - Subfields of Finite Fields
- **Galois Theory** — 10%
    - Fundamental Theorem of Galois Theory
    - Solvability of Polynomials by Radicals

Student Learning Outcomes (SLO): At the end of MTH 512, a student who has studied and learned the material should be able to:

1. Recognize and construct classic examples of rings, integral domains and fields. [PLO: 2,3]
2. Incorporate equivalence relations into ring and field theoretic structures, specifically factor rings. [PLO: 3]
3. Determine subrings/ ideals and determine whether given subsets of a given ring are subrings, ideals or neither. [PLO: 3]
4. Create factor rings using ideals and the First Ring Isomorphism Theorem, and then interpret elements of factor rings accurately. [PLO: 2]
5. Characterize ideals as principal, prime, or maximal and interpret the corresponding significance for the factor ring. [PLO: 3]
6. Construct and manipulate ring homomorphisms and isomorphisms. [PLO: 2]
7. Recognize and interpret theorems to prove properties about specific algebraic structure. [PLO: 1]
8. Refine the skills of proof by contradiction, proof by contraposition, proof of set equality, and proof using both forms mathematical induction. [PLO: 1]
9. Define and test a potential ring or vector space isomorphism for being well-defined, a homomorphism, one-to-one and onto. [PLO: 3]
10. Demonstrate use of the idea of maps that preserve operations and structure in the context of groups, rings, vector spaces and beyond. [PLO: 2,3]
11. Use definitions of one-to-one, onto, well-defined, homomorphism, isomorphism and others to characterize a given map. [PLO: 3]
12. Demonstrate use of the hierarchical structure leading to a field (ID, UFD, PID, ED). [PLO: 3]
13. Master techniques of polynomial ring factorization and irreducibility. [PLO: 2,3]
14. Construct algebraic, transcendental, finite and infinite field extensions. [PLO: 2]
15. Demonstrate use of the interplay between field extensions and the associated automorphism groups which forms the basis for Galois Theory. [PLO: 3]

Program Learning Outcomes (PLO):

Students graduating from SFASU with a M.S. degree and a major in mathematics will:

1. **[*Critical Reasoning*]** Independently apply the principles of logic in mathematics to develop and analyze conjectures and proofs. (understanding of abstract structures, development of definitions, development and proof of conjectures)
2. **[*Skills*]** Execute advanced mathematical procedures and build upon these standard procedures. (learning of new skills, applying or extending skills in new situations)
3. **[*Concepts*]** Demonstrate knowledge of core mathematical concepts. (definitions and theorems in analysis, definitions and theorems in linear or abstract algebra, definitions and theorems in theoretical statistics)

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