STEPHEN F. AUSTIN STATE UNIVERSITY

Department of Mathematics and Statistics

Math 511 – Abstract Algebra I
Course Syllabus

Course description: Groups, subgroups, homomorphisms, isomorphisms, cosets, factor groups, the Fundamental Theorem of Group Homomorphisms, the Fundamental Theorem of Finite Abelian Groups.

Credit hours: 3

Course Prerequisites and Corequisites: MTH 412

Course outline: Approximate time spent

- **Sets & binary operations** 5%
  - Basic set theory
  - Equivalence relations
  - Binary operations
  - Isomorphic structures
- **Introductory Group Theory** 25%
  - Definitions of group and subgroups
    - Canonical examples
    - Subgroup tests
    - Abelian groups
  - Notions of homomorphism and isomorphism
    - Basic definitions
    - Properties
    - Tests for proving homomorphism
    - Tests for proving isomorphism
  - Cyclic groups
    - Fundamental Theorem of Cyclic Groups
  - Generating Sets and Cayley diagrams
- **Permutations, Cosets and Direct Products** 25%
  - Permutation groups
  - Orbits and cycles
  - Alternating groups
  - Cosets
    - Theorem of Lagrange
  - External/ Internal Direct products
  - Finitely generated abelian groups
    - Fundamental Theorem of Finite Abelian Groups
- **Homomorphisms** 15%
  - Definition and intuition of homomorphism
  - Basic properties
  - Tests for proving homomorphism
  - Tests for proving isomorphism
- **Normal Subgroups and Factor Groups** 15%
  - Kernels
  - The First Isomorphism Theorem
- **Sylow Theorems** 10%
  - Cauchy's Theorem
- **[Finite Simple Groups]** 5%

Student Learning Outcomes (SLO): At the end of MTH 511, a student who has studied and learned the material should be able to:

1. Incorporate equivalence relations into group theoretic structures, particularly factor groups. [PLO: 3]
2. Determine subgroups and determine whether given subsets of a group are subgroups. [PLO: 3]
3. Use the Fundamental Theorem of Cyclic Groups to classify and determine subgroup structure of non-cyclic groups. [PLO: 3]
4. Construct and manipulate group homomorphisms and isomorphisms. [PLO: 2]
5. Recognize and interpret theorems to prove properties about specific algebraic structure. [PLO: 1,5]
6. Use the skills of proof by contradiction, proof by contraposition, proof of set equality, and proof using both forms mathematical induction. [PLO: 1,5]
7. Define and test a potential isomorphism for being well-defined, a homomorphism, one-to-one and onto. [PLO: 1,2,4]
8. Use definitions of one-to-one, onto, well-defined, homomorphism, isomorphism and others to characterize a given map. [PLO: 3]
9. Create factor groups using normal subgroups or the First Isomorphism Theorem and interpret elements of factor groups accurately. [PLO: 3,4]
10. Demonstrate understanding of permutations and symmetries in a group theoretic context – particularly the significance of Cayley's Theorem. [PLO: 3]
11. Recognize and use the Sylow Theorems to characterize certain finite groups. [PLO: 3]

Program Learning Outcomes (PLO):

Students graduating from SFASU with a M.S. degree and a major in mathematics will:

1. **[*Critical Reasoning*]** Independently apply the principles of logic in mathematics to develop and analyze conjectures and proofs. (understanding of abstract structures, development of definitions, development and proof of conjectures)
2. **[*Skills*]** Execute advanced mathematical procedures and build upon these standard procedures. (learning of new skills, applying or extending skills in new situations)
3. **[*Concepts*]** Demonstrate knowledge of core mathematical concepts. (definitions and theorems in analysis, definitions and theorems in linear or abstract algebra, definitions and theorems in theoretical statistics)
4. **[*Problem Solving*]** Demonstrate initiative in using various mathematical tools, including technology, to formulate, represent, and solve problems. (implement algorithms or definitions, discuss algorithmic proficiency, find numerical approximations)
5. **[*Communication*]** Demonstrate proficiency in communicating mathematics in a format appropriate to expected audiences. (written, visual, oral)