

**Department of Mathematics and Statistics** 

# MTH 502 – Mathematical Statistics I Course Syllabus

<u>Course description</u>: Random variables, discrete and continuous distributions, multiple random variables, distributions of functions of random variables, convergence concepts.

# Credit hours: 3

## Course Prerequisites and Corequisites: MTH 439 or equivalent

| Course | outline:  | Approximate time spent |
|--------|---|------------------------|
| •      | Probability Functions and Spaces  | 5%                     |
|        | <ul> <li>Review of Set Theory</li> </ul>  |                        |
|        | <ul> <li>Probability Functions &amp; the Axioms</li> </ul>                                |                        |
|        | <ul> <li>The Probability Space</li> </ul>   |                        |
|        | <ul> <li>Elementary Probability Rules Based on the Axioms</li> </ul>                      |                        |
| •      | Results of Conditioning & Independence  |                        |
|        | <ul> <li>Conditional Probability</li> </ul>   | 10%                    |
|        | <ul> <li>Theorem of Total Probabilities</li> </ul>  |                        |
|        | <ul> <li>Bayes' Rule</li> </ul>   |                        |
|        | <ul> <li>Independent Events</li> </ul>  |                        |
| ٠      | Language of Random Variables  | 10%                    |
|        | <ul> <li>Discrete vs. Continuous Random Variables</li> </ul>                              |                        |
|        | <ul> <li>Cumulative Distribution Function</li> </ul>                                      |                        |
|        | <ul> <li>Mass and Density Functions</li> </ul>  |                        |
| ٠      | Summary of Random Variables   | 10%                    |
|        | <ul> <li>Expected Value (Discrete and Continuous)</li> </ul>                              |                        |
|        | <ul> <li>Variance and Standard Deviation of Random Variabl</li> </ul>                     |                        |
|        | <ul> <li>Moment Generating Functions (and other Generating</li> </ul>                     |                        |
| ٠      | Discrete Distribution Theory  | 20%                    |
|        | <ul> <li>Uniform Distributions</li> </ul>   |                        |
|        | • Bernoulli Trials  |                        |
|        | <ul> <li>Binomial Models</li> </ul>   |                        |
|        | Geometric Models  |                        |
|        | <ul> <li>Negative Binomial Models</li> </ul>  |                        |
|        | <ul> <li>Hypergeometric Models</li> <li>Poisson Models and the Poisson Process</li> </ul> |                        |
|        |   |                        |
|        | • Relationships Between Discrete Probability Models                                       | 25%                    |
| •      | Continuous Distribution Theory <ul> <li>Uniform Distributions</li> </ul>                  | 25%                    |
|        |   |                        |
|        | <ul> <li>Exponential and Gamma Models</li> <li>The Normal Distribution</li> </ul>         |                        |
|        | <ul> <li>The Reta and Other Continuous Models</li> </ul>                                  |                        |
|        | <ul> <li>Relationships Between Models</li> </ul>  |                        |
|        | <ul> <li>Exponential and Poisson</li> </ul>   |                        |
|        | <ul> <li>Gamma and Poisson</li> </ul>   |                        |
|        | <ul> <li>Uniform and Exponential</li> </ul>   |                        |
|        |   |                        |

• Truncation and Mixtures of Random Variables

MTH 502 – Mathematical Statistics I Syllabus Continuation

#### Multivariate Probability Models

- o Joint Cumulative Distribution Functions
- Joint Mass Functions
  - The Multinomial Distribution
- Joint Density Functions
  - The Multivariate Normal Distribution
- Multivariate Expectation
- Conditional Distributions
  - Independent Random Variables
  - Conditional Expectation and Variance
  - Double Expectation Theorem
- Covariance and Correlation

## • Transformations of Random Variables

- The Cumulative Distribution Function Method
- The Moment Generating Function Method
- Transformation Theorems

<u>Student Learning Outcomes (SLO)</u>: At the end of MTH 502, a student who has studied and learned the material should be able to:

- 1. Apply the axioms of probability and basic probability laws in order to compute likelihood of events in various scenarios. [MTH-PLO: 3,4], [STA-PLO: 1]
- 2. Recognize when conditional probabilities are relevant and be able to calculate a variety of conditional probabilities using several techniques. [MTH-PLO: 2, 4], [STA-PLO: 1]
- 3. Explain the need for summarizing random variables and successfully compute the expected value and standard deviation of random variables useful in practice. [MTH-PLO: 2, 3, 4], [STA-PLO: 1, 4]
- 4. Explain the role and meaning of random variable. [MTH-PLO: 3, 5], [STA-PLO: 1, 4]
- 5. Model random natural phenomena using discrete and continuous probability distributions. [MTH-PLO: 2, 4], [STA-PLO: 2]
- 6. Explain the relationships which exist between the major probability distributions. [MTH-PLO: 3, 5], [STA-PLO: 2, 5]
- 7. List the main features of the popular discrete and continuous probability models. [MTH-PLO: 3], [STA-PLO: 1, 2]
- 8. Calculate probabilities in higher dimensions and model multivariate random variables. [MTH-PLO: 2, 4], [STA-PLO: 1]
- 9. Explain the need for functions of random variables and determine the appropriate density function for the function of a continuous random variable. [MTH-PLO: 4, 5], [STA-PLO: 1, 5]
- 10. Delineate between the major methods useful for finding the distribution of a function of random variables. [MTH-PLO: 2, 4], [STA-PLO: 1, 5]
- 11. Calculate and explain the relevance of correlation and its interpretation. [MTH-PLO: 2, 5], [STA-PLO: 1, 5]

15%

5%

MTH 502 – Mathematical Statistics I Syllabus continuation

### Program Learning Outcomes (MTH - PLO):

Students graduating from SFASU with a M.S. degree and a major in mathematics will:

- 2. **[Skills]** Execute advanced mathematical procedures and build upon these standard procedures. (learning of new skills, applying or extending skills in new situations)
- 3. **[Concepts]** Demonstrate knowledge of core mathematical concepts. (definitions and theorems in analysis, definitions and theorems in linear or abstract algebra, definitions and theorems in theoretical statistics)
- 4. **[Problem Solving]** Demonstrate initiative in using various mathematical tools, including technology, to formulate, represent, and solve problems. (implement algorithms or definitions, discuss algorithmic proficiency, find numerical approximations)
- 5. **[Communication]** Demonstrate proficiency in communicating mathematics in a format appropriate to expected audiences. (written, visual, oral)

#### Program Learning Outcomes (STA - PLO):

Students graduating from SFASU with an M.S. degree and a major in statistics will demonstrate:

- 1. A command of core probability and statistical concepts through major definitions and theorems. [Concepts] (Probability and Statistical Inference)
- 2. Strategic competence in formulating a standard probabilistic/statistical model for a given problem. [*Modeling*] (Model Choice and Model Interpretation)
- The ability to independently apply principles of probability and statistics to model and solve new or non-standard problems. [Independent Thinking and Application] (Existing Literature Comprehension, Independent Progression, Resourcefulness)
- 5. Proficiency in communicating probability and statistics in a format appropriate to expected audiences. **[Communication]** (Written Communication, Oral Communication)

Date of document: 04/01/2009